TONY BOMFORD'S SEVENTEEN HAND-KNOTTED RUGS

Made in Canberra, Australia 1974 - 1988

Tony Bomford's Seventeen Hand-knotted Rugs

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1988.

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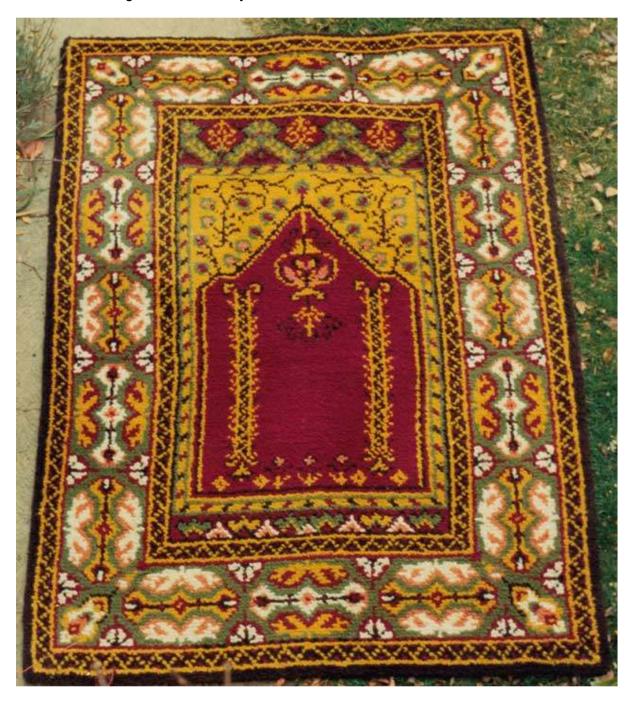
RUG ONE

Rectangular runner, 36 by 108 inches, 0.9144 by 2.7432 m; 43,200 knots Started 1974 May 4; finished 1974 Aug 4 Inscribed "Bomford 1974" in Morse code in the top and bottom border Commercial design "Kabistan 2295" by Paton & Baldwin



RUG TWO

Rectangular, 48 by 68 inches, 1.219 m by 1,727 m; 36,480 knots Started 1974 Sep 24; finished 1975 Mar 1 Inscribed in Morse code on the inner line of the outer band, "Tony Bomford 1975" Commercial design "Milas 2047" by Paton & Baldwin



RUG THREE

Rectangular rug 1.59 by 2.39 m, 38,001 knots Started 1975 Mar 9; finished 1976 Jan 10 Inscription in Morse code in the outer blue line of the border The first rug I designed myself, feeling my way into a new medium - still with a long way to go.



RUG FOUR

Rectangular rug knotted on the cross, 2.07 by 1.55 m; 55,296 knots Started 1975 Dec 26; finished 11 May 1976

Inscribed in Morse code in the SW and SE corner diagonals TONY BOMFORD DESIGNED AND MADE THIS RUG FOR ELIZABETH BOMFORD 1976

Translated from a signed mosaic by L'oeuf on Le Tour Belledonne, one of Les Trois Tours de Grenoble, tall apartment blocks.



Tony Bomford's RUG ELEVEN

Rectangular runner, 6 ft 7 in by 2 ft 1 in; 26,500 knots

Started 1981 May 30; finished 1981 Nov 22

Boldly inscribed: AGB 1981

This was my only attempt at design-as-you go. Rug 11 was initially intended to depict a nude, but the difficulty of the foreshortened toes and feet proved severe and it was foreseen that the difficulties of the naked human body would eventually prove insuperable.

It is a matter of opinion whether the anklets, garters, legs, knees, kilt, knitted pullover and wig proved manageable or not; nor is it clear to me why the two central sections were diminished in proportion to the top and bottom sections. The domestic animals include Black Catty, Mitty Bom II and Jessie, at that time our golden retriever. The wildlife includes a Pelican, a Gannet and a Sea-eagle.

It served me well in its time as a yoga mat.



Tony Bomford's RUG FIVE

Rectangular rug, knotted on the cross, 2.03 by 1.31 m; 46,986 knots Started 1976 May 30; finished 1977 Jan 2 Inscribed in the border BOMFORD 1976 SCI AMER DEC 75

This was my first rug to utilize Richard E James III's new tessellation of convex pentagons discovered in 1975 and reported in the Scientific American, Dec 1975, page 116

This rug was used as Plate 5, an illustration in *The Mathematical Gardner*, edited by David A. Klarner and published in 1981, a tribute to Martin Gardner, editor of the Mathematical Games column in the Scientific American for many years.



RUG SIX

Rectangular rug, knotted on the cross, 2.51 by 1.57 m; 68,200 knots Started 19?? Feb 10; finished 1977 Dec 9

Cryptically inscribed A G BOMFORD 1977 halfway down the middle of the outer edge of the inner bands of the sides

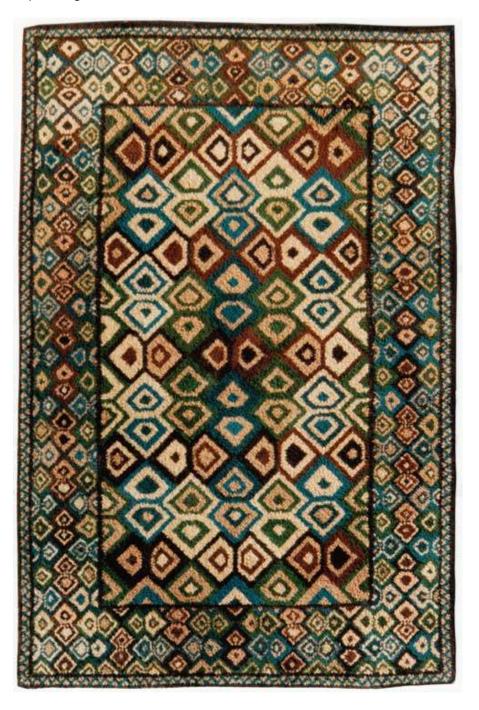
My second rug based on Richard E James's new tessellation of convex pentagons is similar to the first but the symmetry has been made more obvious by lines of brown pentagons down the borders and the five rows of octagons and five brown lines across the field. In all my Richard James rugs, the pentagons in the central field are all different. In the border, the number of available pentagons is reduced owing to their being smaller, and a diligent search may detect a few duplicates.



RUG NINE

Rectangular, knotted on the cross, 2.18 by 1.43 m. 53,998 knots Started 1980 Jan 26; finished 1980 Sep 10 Inscribed BOMFORD 1980 SCI AMER 1975

This was my third and last rug based on Richard E James's discovery of a sixth way of tiling the plane with convex pentagons. It is readily distinguished from the others by the brown line separating the field from the border.



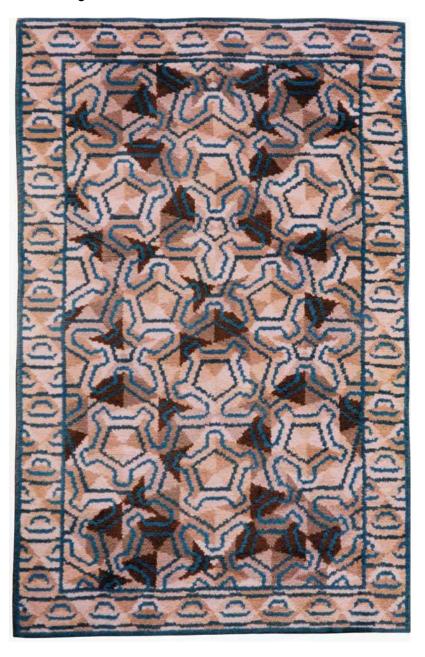
Tony Bomford's RUG SEVEN

Rectangular, 7 ft 4 in by 5 ft; 64,800 knots

Started 1977 Dec 17; finished 1978 Jul 14

Inscribed: A G BOMFORD 1978 PENROSE SCI AMER NOV 1977

This was my first rug with the newly discovered kites and darts tiling the plane in a non-periodic manner. I have converted Conway's circles into pentagons, but they still have straight sides. Conway proved that in any two-dimensional Penrose universe two lines, at most, one of each colour, enter the pattern from the outer edge, cross the central decagon and disappear off to infinity. Can you find them on this and the other three rugs? In the surrounding border, the kites and darts are here arranged in a periodic way, Conway's lines not meeting.



Tony Bomford's RUG EIGHT

Large rectangular rug, 10 ft 8 in by 8 ft 8 in; 145,856 knots Started 1978 Aug 29; finished 1980 Jan 24 Cryptically inscribed on the outer line of the two-line inner band separating the field from the border, starting from the top and reading from the back:

TONY BOMFORD DESIGNED AND MADE THIS RUG 1978-1980 PENROSE NON-PERIODIC TILING SCI AMER JAN 77

This was my most ambitious rug based on the newly discovered non-periodic tiling of kites and darts. The green lines on the kites and darts change to blue above the horizon outside the central area to increase the sense of a pastoral design. Once the central pattern of twelve dark green half-darts and two whole darts was chosen the rest of the pattern was constrained outward into space. Nearly all the lines form loops, but as in most Penrose non-periodic patterns two lines, one light green, one dark green, reach out into infinite space. The grey colours were chosen to reveal the crude decagonal symmetry, but the kites and darts are only symmetrical east-west, and only in shape, not in colour.

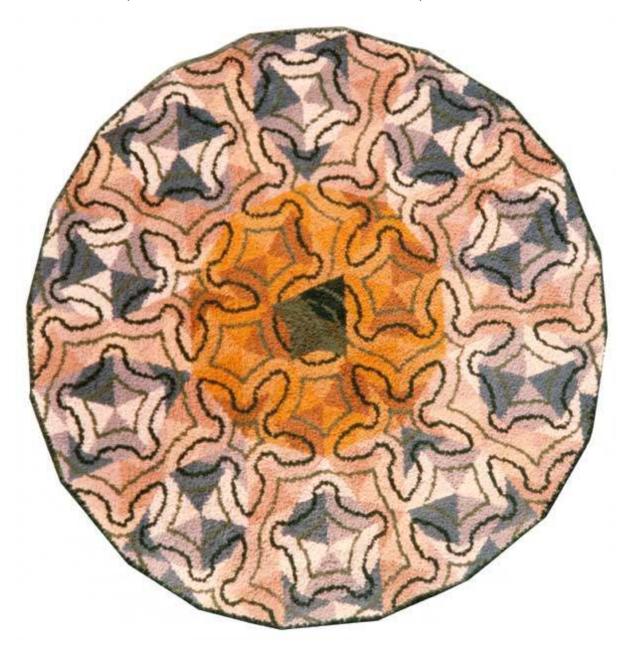
When the lines connect end to end the kites and darts form a non-periodic tessellation. The horizontal and vertical borders show the standard, rather dull, periodic tessellation obtained when the kites and darts join to form golden rhombs, "golden" referring to their proportion, not their colour.



Tony Bomford's RUG TEN

Icosigon with twenty sides, 6 ft 8 in between opposite sides; 55,572 knots Started 1980 Sep 10; finished 1981 Apr 23

This apparently simple rug based on the non-periodic tiling of kites and darts seems at first to be symmetrical, apart from colours, but it has no axis of symmetry. The arrangement of twelve half-darts in the centre determines the rest of the pattern. Two lines, one of each colour, cross the pattern from side to side. All the rest form loops.



Tony Bomford's RUG FOURTEEN

Rectangular, 2.67 by 2.10 m; 96,600 knots exactly Started 1984 Mar I; finished 1985 Mar 20, and whimsically entitled, *Sun giving birth to planets*.

The central part of a two-dimensional Penrose universe non-periodically tiled with kites and darts cut from a rhombus so as to divide the long diagonal in the golden ratio. The borders along the short sides show the kites and darts arranged in the normal periodic manner. The red lines on each kite and dart are so chosen that when their ends connect, the pattern is non-periodic. This particular pattern, which depends on the arrangement at the centre, is unusual in that, colours apart, it has radial symmetry, and all the lines form loops, none runs off to outer space.

When moved in 2003, it was found that water had dissolved the glue in the 'canvas' to which the wool is knotted, leaving a hole, and the rug needs repair.



Tony Bomford's RUG TWELVE

Circular, 2.18 m in diameter, 64,242 knots Started 1981 Dec 23; finished 1982 Oct 19

This was my first rug showing a tessellation on the infinite hyperbolic plane conformally projected on to the Euclidean plane. It shows hexagons meeting four at a point. This is only possible on the hyperbolic plane: on the Euclidean plane hexagons meet three at a point. When conformally projected on to the Euclidean plane, the entire infinite hyperbolic plane is bounded by a circle of finite radius, and straight lines on the hyperbolic plane project as axes of circles which meet the bounding circle at right angles, as can readily be seen here.

Exhibited in Perth, WA, in 1985 at the first meeting of the Congress of the International Cartographic Association in Australia.



RUG THIRTEEN

Circular, diameter 2.18m; 66,252 knots Started 1982 Oct 24; finished 1983 Nov 11

This is my second rug based on hexagons meeting four at a point on the hyperbolic plane and conformally projected on to the Euclidean plane. It is the most ambitious rug I have attempted, the colours being jumbled as if seen in a vibrated dish of shallow water. Owing to the great range of colour from near white to dark blue the rug has the curious property of looking more pleasing when observed directly than when seen in photographs

Exhibited in Perth in 1985 at the first meeting of the Congress of the International Cartographic Association in Australia

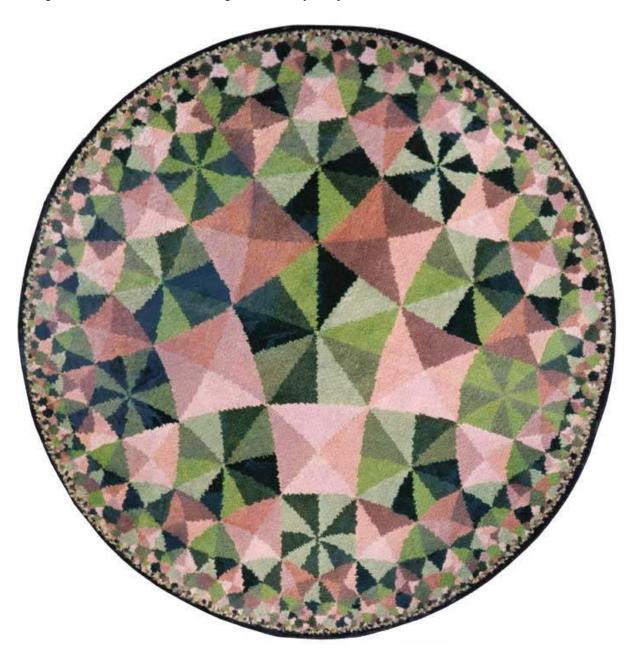


Tony Bomford's RUG FIFTEEN

Circular, diameter 2.184 m;64,242 knots Started 1985 Mar 20; finished 1986 Feb 11 Cryptically inscribed in Morse code around the outer border:

TO THE GLORY OF GOD AND TO H S M COXETER THIS RUG IS DEDICATED BY TONY BOMFORD WHO MADE IT 1988

This is a conformal projection of the quasi-regular tessellation of squares and pentagons from the hyperbolic plane on to the Euclidean plane. The squares are brown, the pentagons green, and they meet alternately four at a point. The simple tessellation of regular pentagons meeting four at a point can be seen by augmenting the green pentagons with a brown triangle, one of each colour, on each side; and the simple tessellation of squares meeting five at a point can be seen by augmenting the brown squares with a two-coloured green triangle on each side, the same greens always adjacent to the same browns.

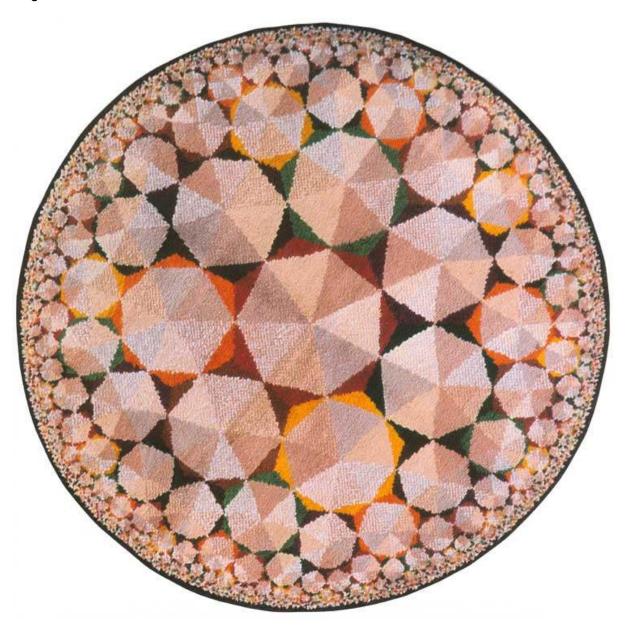


Tony Bomford's RUG SIXTEEN

Circular, diameter 7 ft; 62,575 knots Started 1986 Jul 11; finished 1987 Jul 25

This is the first of two rugs based on a conformal projection of a quasi-regular tessellation of heptagons and triangles from the hyperbolic plane on to the Euclidean plane. The seven triangles in each heptagon are coloured in one of eight different shades of brown, four of which are "tweeds" with alternate knots of different shades. The triangles are divided into thirds, each third coloured in one of eight bright primary or secondary colours which can be distinguished more readily on the rug than in a photograph.

Rug 16 reveals six more tessellations which are described and illustrated in an annex.



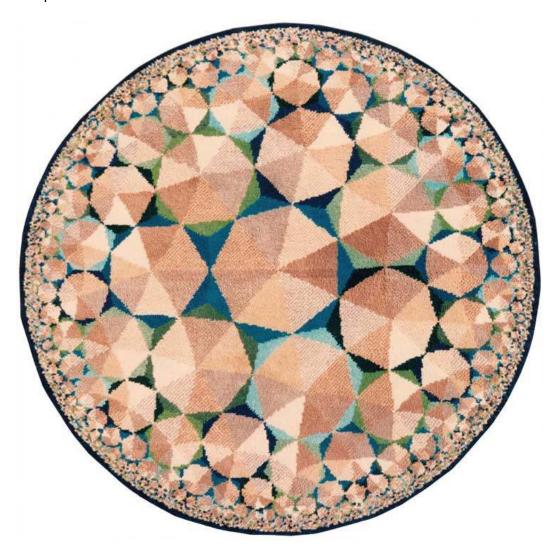
Tony Bomford's RUG SEVENTEEN

Circular, diameter 7 ft; 62,575 knots Started 1987 Jul 26; finished 1988 Oct 12 Inscribed on the inner ring of the outer border:

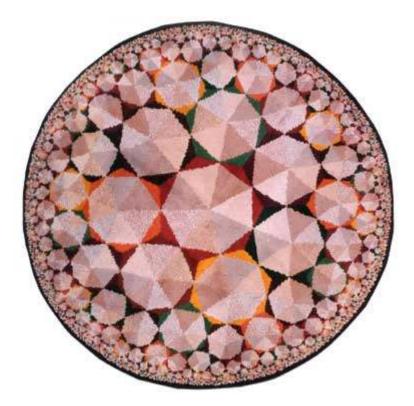
TONY BOMFORD CREATED THIS RUG FOR SUSAN COX 1988

This is the second of two rugs based on a conformal projection of a quasi-regular tessellation of heptagons and triangles from the hyperbolic plane on to the Euclidean plane. The seven triangles in each heptagon are coloured in one of eight different shades of brown, four of which are "tweeds" with alternate knots of different shades. The triangles are divided into thirds, each third coloured with one of eight shades of blue or green which can be distinguished more readily on the rug than in photographs. Like Rug 16, from which it differs only in colour, not in pattern or size, Rug 17 reveals six more tessellations which are described and illustrated in an annex.

Rug 17 was give to Susan Cox, companion from 1985 to 1998 on most of my best and hardest journeys. She took it back to America with her in 1993, but her plane was delayed and she had a tight connection in Los Angeles. She got the rug through customs, but her baggage missed her plane to Portland, Oregon, and next morning only her suitcase was delivered. The rug has not been seen again and American Airlines paid \$US 640 compensation.



RUG 16: MY FIRST RUG COVERED IN TRIANGLES AND HEPTAGONS



Rug 16 is covered with equilateral triangles and regular heptagons, without gaps or overlaps, in the ratio of seven triangles to three heptagons.

The triangles are divided into thirds, each third a different, bright colour. The heptagons are divided into sevenths, each seventh a different shade of brown. There are eight "colours" and eight "browns", four of which are "tweeds", with alternate knots in alternate shades; only four shades of brown wool were used. Bright and distinct colours were chosen, to help one find one's way around the pattern, especially where it gets smaller towards the edge.

The triangles look like this:



The brown heptagons look like this:



1. Rug 16 has polygons of two kinds, both regular, which meet alternately at every vertex, and cover the rug without gaps or overlaps. The pattern is called a "quasi-regular tessellation" and is depicted symbolically by [3:7].

Besides the [3:7], Rug 16 depicts six more related tessellations:

2. Larger triangles cover the rug and meet seven at a point to form a [3,7]. Each consists of four equilateral triangles, the three corner triangles the same shade of brown, the central triangle divided into three congruent triangles of different bright colours.



Since there are only 336 possible combinations of 8 colours taken 3 at a time, identical three-coloured central triangles exist, always lying in a triangle of the same shade of brown.

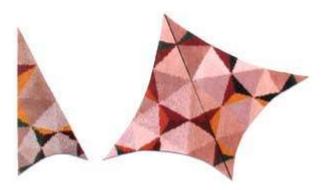
3. Slightly larger heptagons meet three at a point. Each has a brown centre with a uniformly coloured border. This [7,3] tessellation is the dual of the [3,7]. The sides of dual tessellations bisect each other at right angles, as tan be seen with this pair. The heptagon surrounded by red triangles in the centre of the rug is surrounded by seven others surrounded by triangles coloured yellow, chestnut, orange, dark brown, light green, purple and dark green blue, bright green, dark blue, pale green, dark green, pale blue and mid green.





Further out is a ring of fourteen heptagons surrounded by red triangles, centred on the seven lines which bisect the central hexagon, extended in both directions. Every other hexagon is ringed by fourteen hexagons of its own colour, but most are too small to be seen on the rug.

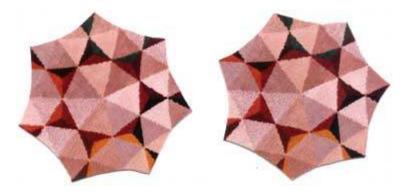
- 4. Larger heptagons meet at the centre of every [7,3] heptagon seven times in two rotations, giving rise to the mathematical notation of [7,7/2]. Each triangle occurs in three different heptagons, so this tessellation covers the plane three times over, and is said to have a density of three.
- 5. For brevity, let us call a heptagon surrounded by, say, red triangles, a "red heptagon". Then larger equilateral triangles have vertices at the centres of any three adjacent heptagons of the same colour: such as the centre of the central red heptagon and the centres of any two adjacent red heptagons of the fourteen that surround it. Fourteen of these triangles meet at the centre of each heptagon, so their angles are $\pi/7$, and the tessellation is a [3,14]. These triangles may not appear equilateral, but their sides as well as their angles are all alike, as can be seen more clearly from the pair of triangles with vertices in the red heptagons: around a triangle, the pattern along each side is the same.



6. 14-gons are formed by the fourteen triangles of the previous sort which meet at the centre of a heptagon. The vertex angles are twice that of a triangle, or $2\pi/7$, so seven meet at a point and the tessellation is a [14,7].



7. Still larger heptagons are centred on each brown heptagon with vertices at the common vertex of a pair of triangles. The vertices are right angles, and the tessellation is therefore a [7,4]. According to the coloured triangles chosen, the bisected pieces may point either clockwise or anticlockwise.



Each of the eight bright colours is paired with one of the eight shades of brown: the shade that does not occur within the heptagons that it surrounds. For example, the red surrounding the central [3:7] and [3,7] in tessellations 1 & 2 is paired with a pale tweeded beige, the colour that does on appear within those heptagons: yet all the heptagons bordered with other colours contain triangles of that colour.

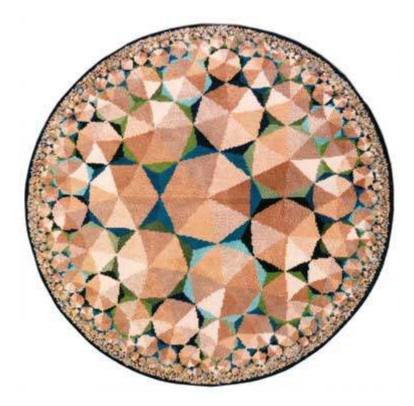
The inner ring of pale beige triangles all have their inner vertices pointing clockwise. Thus while the quasi-regular tessellation on which Rug 16 is based has seven-fold rotational symmetry, Rug 16 itself, as coloured, has not. It could be distinguished at a glance from a mirror image of itself, in which the inner ring of pale beige triangles pointed anti-clockwise. This so- called "enantiomorphous" pattern can be seen by turning Rug 16 over and looking at the back.

On our familiar "Euclidean" plane, tessellations 1 to 7 are impossible: combinations of equilateral triangles and regular heptagons just do not fit together without gaps or overlaps; nor do heptagons by themselves. That is why on Rug 16 the sides of the polygons are not straight and why the polygons get smaller towards the edge. Nor can the tessellations be drawn on the surface of an ordinary sphere: they can be drawn only on the hyperbolic plane, which is the surface of a sphere of radius *i*, the square root of -1.

To most of us this remark is more confusing than helpful because the hyperbolic plane does not lie in our familiar Euclidean 3-space. But mathematicians have explored its mathematical properties, and just as an atlas maker can project any design drawn upon the surface of a sphere or spheroid so that angles or distances (but not both) are not changed, so we can project the hyperbolic plane on to the Euclidean plane preserving angles, but not distances. The projection has two pleasing properties: the entire, infinite hyperbolic plane projects within a bounding circle of finite radius - the edge of our rugs; and straight lines on the hyperbolic plane project as arcs of circles that meet the bounding circle at right-angles, as can frequently be seen. On Rug 16 the quasi-regular tessellation of equilateral triangles and regular heptagons on the hyperbolic plane has been projected on to the Euclidean plane without distorting angles. On the hyperbolic plane itself, not only are the angles of the equilateral triangles and regular heptagons all equal, but their sides also are all equal, and all straight.

Tony Bomford 2003 April 20

RUG 17: MY SECOND RUG COVERED IN TRIANGLES AND HEPTAGONS



Rug 17 is covered with equilateral triangles and regular heptagons, without gaps or overlaps, in the ratio of seven triangles to three heptagons. It is identical to Rug '16 except for colour.

The triangles are divided into thirds, each third a different shade of blue or green. The heptagons are divided into sevenths, each seventh a different shade of brown. There are eight "colours" and eight "browns", four of which are "tweeds", with alternate knots in alternate shades; only four shades of brown wool were used. Bright and distinct shades of blue and green were chosen, to help one find one's way around the pattern, especially where it gets smaller towards the edge.

The triangles look like this:



The brown heptagons look like this:



1. Rug 17 has polygons of two kinds, both regular, which meet alternately at every vertex, and cover the rug without gaps or overlaps. The pattern is called a "quasi-regular tessellation" and is depicted symbolically by [3:7].

Besides the [3:7], Rug 17 depicts six more tessellations:

2. Larger triangles cover the rug and meet seven at a point to form a [3,7]. Each consists of four equilateral triangles, the three corner triangles the same shade of brown, the central triangle divided into three congruent triangles of different shades of blue or green. Since there are only 336 possible combinations of 8 colours taken 3 at a time, identical three-coloured central triangles exist, always lying in a triangle of the same shade of brown.



3. Slightly larger heptagons meet three at a point. Each has a brown centre surrounded by triangles of the same colour. This [7,3] tessellation is the dual of the [3,7]. The sides of dual tessellations bisect each other at right angles, as can be seen with this pair. The heptagon surrounded by bright blue triangles in the centre of the rug is surrounded by seven other heptagons surrounded by triangles coloured mid blue, bright green, dark blue, pale green, dark green, pale blue and mid green. Further out is a ring of fourteen heptagons surrounded by bright blue triangles, centred on the seven lines which bisect the central hexagon, extended in both directions. Every other hexagon is ringed by fourteen hexagons of its own colour, but most are too small to be seen on the rug.

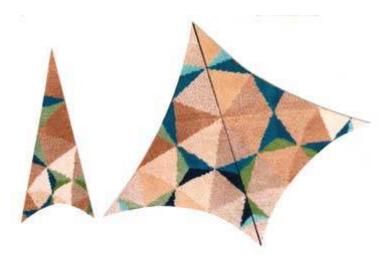


4. Larger heptagons meet at the centre of every [7,3] heptagon seven times in two rotations, giving rise to the mathematical notation of [7,7/2]. Each triangle occurs in three different heptagons, so this tessellation covers the plane three times over, and is said to have a density of three.

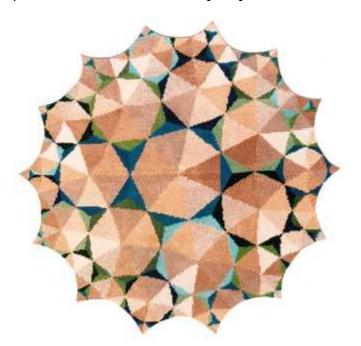


5. For brevity, let us call a heptagon surrounded by, say, light blue triangles a light blue heptagon. Then larger equilateral triangles have vertices at the centres of any three adjacent heptagons of the same colour; such as the centre of the central bright blue heptagon and the centres of two adjacent bright blue heptagons of the fourteen that surround it. Fourteen of

these triangles meet at the centre of each heptagon, so their angles are $\pi/7$, and the tessellation is a [3,14]. These triangles may not appear equilateral, but their sides as well as their angles are all alike, as can be seen from a pair of triangles with vertices in the pale blue heptagons: around a triangle, the pattern along each side is the same.



6. 14-gons are formed by the fourteen triangles of the previous sort which meet at the centre of a heptagon. The vertex angles are twice that of a triangle, or $2\pi/7$, so seven meet at a point and the tessellation is a [14,7].



7. Still larger heptagons than tessellation 4 are centred on each brown heptagon, with vertices at the common vertex of a pair of triangles. The vertices are right angles, and the tessellation is therefore a [7,4]. According to the coloured triangles chosen, the bisected pieces may point either clockwise or anticlockwise.



Each of the eight bright colours is paired with one of the eight shades of brown: the shade that does not occur within the hexagon. For example, the bright blue surrounding the central [7:3] - tessellation 4 - is paired with white, the colour that does on appear in tessellation 4 or 2; yet all the heptagons bordered with other colours contain white triangles.

The inner ring of large white triangles all have their inner vertices pointing anti-clockwise. Thus while the quasi-regular tessellation on which Rug 17 is based has seven-fold rotational symmetry, Rug 17 itself, as coloured, has not. It is said to be "enantiomorphous" and can be distinguished at a glance from a mirror image of itself, in which the inner ring of white triangles pointed clockwise, which can be seen by turning Rug 17 over and looking at the back.

On our familiar "Euclidean" plane, tessellations 1 to 6 are impossible: combinations of equilateral triangles and regular heptagons just will not fit together without gaps or overlaps; nor will heptagons by themselves. That is why on Rug 17 the sides of the polygons are not straight - they are arcs of circles which meet the bounding circle at right angles - and why the polygons get smaller towards the edge. Nor can the tessellations be drawn on the surface-of an ordinary sphere: they can be drawn only on the hyperbolic plane, the surface of a sphere of radius *i*, the square root of -1.

To most of us this remark is more confusing than helpful because the hyperbolic plane does not lie in our familiar Euclidean 3-space. But mathematicians have explored its mathematical properties, and just as an atlas maker can project any design drawn upon the surface of a sphere or spheroid so that angles or distances (but not both) are not changed, so we can project the hyperbolic plane on to the Euclidean plane preserving angles, but not distances. The projection has two pleasing properties: the entire, infinite hyperbolic plane projects within a bounding circle of finite radius - the edge of our rugs; and straight lines on the hyperbolic plane project as arcs of circles that meet the bounding circle at right-angles, as can frequently be seen. On Rug 17 the quasi-regular tessellation of equilateral triangles and regular heptagons on the hyperbolic plane has been projected on to the Euclidean plane without distorting angles. On the hyperbolic plane itself, not only are the sides and angles of the equilateral triangles and regular heptagons are all equal, but their sides also are all equal, and all straight.

Tony Bomford 2003 April 20